Chapter 7
Solution of Triangles

7.1 Solution of Triangles:

A triangle has six parts in which three angles usually denoted by α, β, γ and the three sides opposite to α, β, γ denoted by a, b, c respectively. These are called the elements of Triangle. If any three out of six elements at least one side are given them the remaining three elements can be determined by the use of trigonometric functions and their tables.

This process of finding the elements of triangle is called the solution of the triangle.

First we discuss the solution of right angled triangles i.e. triangles which have one angle given equal to a right angle.

In solving right angled triangle γ denotes the right angle. We shall use the following cases

Case-I:

When the hypotenuse and one Side is given.

Let a & c be the given side and hypotenuse respectively. Then angle α can be found by the relation.

Sin $\alpha = \frac{a}{c}$

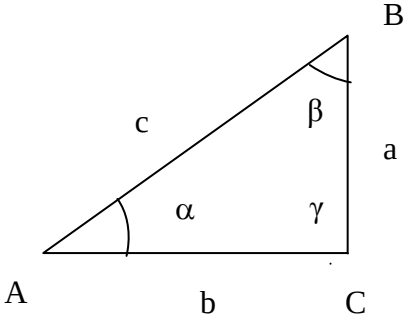


Fig.7.1

Also angle β and side "b" can be obtained by the relations

$\beta = 90^o - \alpha$ and Cos $\alpha = \frac{b}{c}$

Case-II:

When the two sides a and b are given. Here we use the following relations to find α , β & c.

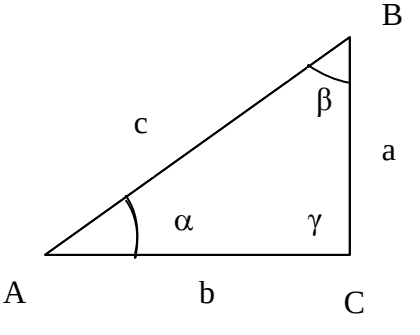


Fig.7.2

Tan $\alpha = \frac{a}{b}$, $\beta = 90^o - \alpha$,

$c = \sqrt{a^2 + b^2}$

Case-III:

When an angle α and one of the sides b , is given. The sides a , c and β are found

from the following relations.

Tan $\alpha = \frac{a}{b}$ and Cos $\alpha = \frac{b}{c}$, $\beta = 90^o - \alpha$

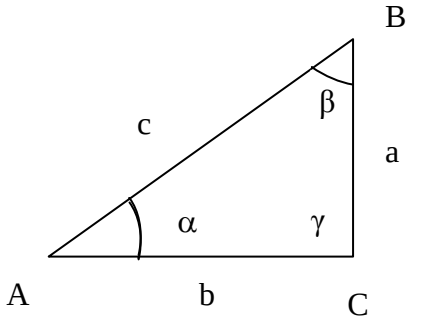


Fig.7.3

Case-IV:

When an angle α and the hypotenuse 'c' is given.The sides a , b and β can be found from the following relations.

Sin $\alpha = \frac{a}{c}$, Cos $\alpha = \frac{b}{c}$ and $\beta = 90^o - \alpha$

Example-1:

Solve the right triangle ABC in which $\alpha = 34^o\ 17'$, b=31.75, $\gamma = 90^o$

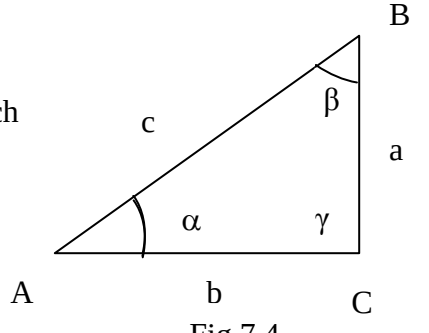


Fig.7.4

Solution:

Given that

$\alpha = 34^o\ 17'$, b = 31.75, $\gamma = 90^o$

We have to find

a = ? c = ? β = ?

Tan $\alpha = \frac{a}{b}$

Tan $34^o\ 17' = \frac{a}{31.75}$

$\Rightarrow$ a = 31.75 tan $34^o\ 17'$

a = 31.75 (0.6817) = 21.64

Also $\cos\alpha = \frac{b}{c}$

$\Rightarrow$ Cos $34^o\ 17' = \frac{31.75}{c}$

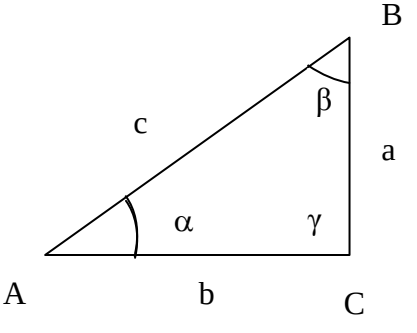


Fig.7.5

$C = \frac{31.75}{\text{Cos } 34^o\ 17'}$ $\Rightarrow$ $\beta = 90^o - 34^o\ 17' = 55^o\ 43'$

Example 2:

Solve the right ΔABC in which $\gamma = 90^o$, a = 450, b = 340

Solution:

$a = 450, \quad b = 340, \quad \gamma = 90^o,$

$c = ? \ = \alpha = ? \ \beta = ?$

$\text{Tan}\,\alpha = \frac{a}{b}$

$\text{Tan}\,\alpha = \frac{450}{340} = 1.3231$

$\Rightarrow \quad \alpha = 52^o\ 56'$

$\beta = 90^o - \alpha = 90^o - 52^o\ 56' = 37^o\ 4'$

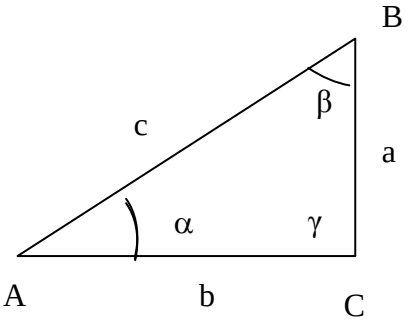


Fig. 7.6

By Pythagoras theorem:

$C^2 = a^2 + b^2 = (450)^2 + (340)^2 = 318100$

$C = 564$

Exercise 7.1

Solve the right triangle ABC in which $\gamma = 90^o$

(1) $a = 250, \quad \alpha = 42^o\ 25'$ (2) $a = 482, \quad \alpha = 35^o\ 36'$

(3) $a = 5, \quad c = 13$ (4) $b = 312, \quad \alpha = 23^o\ 42'$

(5) $a = 212, \quad \beta = 40^o\ 55'$ (6) $c = 232, \quad \beta = 52^o\ 46'$

(7) $c = 540, \quad a = 380$

Answers 7.1

1. $\beta = 47^o\ 35' \quad b = 273.63 \quad c = 370.64$
2. $\beta = 54^o\ 24' \quad b = 673.25 \quad c = 828.01$
3. $b = 12 \quad \alpha = 22^o\ 37' \quad \beta = 67^o\ 23'$
4. $a = 136.96 \quad c = 340.72 \quad \beta = 66^o\ 18'$
5. $\alpha = 49^o\ 05' \quad b = 183.74 \quad c = 280.5$
6. $a = 184.72 \quad b = 140.37 \quad \alpha = 37^o\ 14'$
7. $b = 383.61 \quad \alpha = 44^o\ 44' \quad \beta = 45^o\ 16'$

7.2 Application of Right Angled Triangles

(Measurement of Heights and Distances)

Sometimes we deal with problems in which we have to find heights and distances of inaccessible objects.

The solution of these problems are generally the same as that of solving the right triangles.

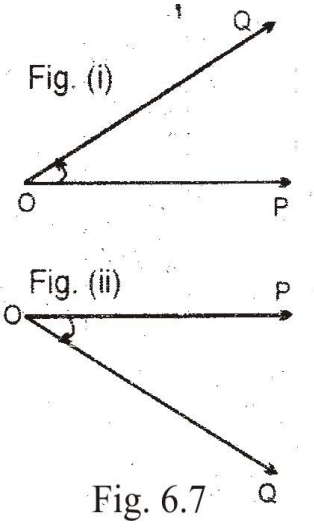


Fig. 6.7

7.3 Angle of Elevation and Depression:

If O be the eye of the observer, Q the position of the object and OP a horizontal line through O then:

i. If Q be above OP, then <POQ is called angle of elevation is shown in Figure (1)

ii. If Q be below OP, then <POQ is called angle of depression is shown in Figure (2)

Example 1:

Find the distance of man from the foot of tower 100m high if the angle of elevation of its top as observed by the man is $52^o\ 30'$.

Solution:

Let, A be the position of man and B be the foot of tower BC. Height of tower = BC = 100m in right Δ AB

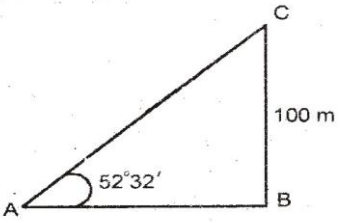


Fig. 6.8

$$\text{Tan } 52^o\ 32' = \frac{BC}{AB}$$

$$1.3032 = \frac{100}{AB} \Rightarrow AB = \frac{100}{1.3032} = 78.73m$$

AB = distance of man from the foot of tower = 76.73m

Example 2:

From the two successive positions on the straight road 1000 meters apart man observes that the angle of elevation of the top a directly ahead of him are of $12^o\ 10'$ and $42^o\ 35'$. How high is the tower above the road.

Solution:

Let, A and D be the two successive positions of a man on the road.

AD = 1000m (Given)

Let BC = height of tower = h = ?

And DB = xm

In Δ ABC

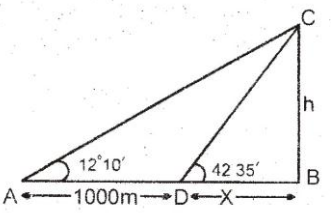


Fig. 6.9

Fig. 7.9

$$\tan 12^o\ 10' = \frac{BC}{AB}$$

$$0.2156 = \frac{h}{(x + 1000)}$$

$$h = 0.2156\ (x + 1000) \ldots\ldots\ldots\ldots(1)$$

In Δ DBC

$$\tan 42^o\ 35' = \frac{BC}{DB} = \frac{h}{x}$$

$$0.9190 = \frac{h}{x}$$

$$x = \frac{h}{0.9190}$$

Put in (1)

$$h = 0.2156\left[\frac{h}{0.9190} + 100\right]$$

$$h = \frac{0.2156}{0.9190}h + \frac{(0,2156)(100)}{0.9190}$$

$$h = 0.2346h + 215.6$$

$$h = 0.2346h = 2156$$

$$0.7654h = 215.6$$

$$h = \frac{2156}{.7654} = 28.168$$

Example 3:

Measure of the angle of elevation of the top of a flag staff observed from a point 200 meters from its foot is :

Solution:

Let height of flog staff = BC = h = ?

A = point of observation

In right Δ ABC

$$\tan 30^o = \frac{h}{200} \Rightarrow \quad h = 200\ (0.577)$$

$$h = 115.4m$$

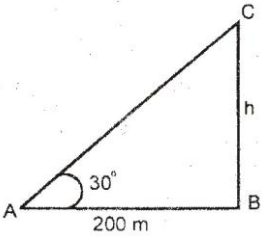


Fig. 6.10

Example 4:

Find the measures of the angle of elevation of the top of a tree 400 meters high, when observed from a point 250 meters away from the foot of the base.

Solution: Given that:

Height of tree = BC = 400m

AB = 250m

Let

$\angle BAC = \alpha$ = ?

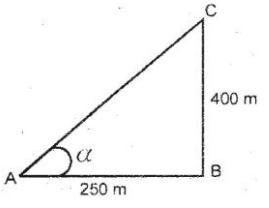


Fig. 6.11

$\angle BAC$= angle of elevation of top of the tree

$$\tan\alpha = \frac{BC}{AB} = \frac{400}{250} = 1.6$$

$$\alpha = \tan^{-1}(1.6) = 58^o$$

Example 5:

The measure of the angle of depression of an airport as observed by a pilot while flying at a height of 5000 meters is $40^o\ 32'$. How far is the pilot from a point directly over the airport?

Solution:

The pilot is at the height of C

BC = 5000m

From right $\angle ABC$

$\tan 40^o\ 32' = \dfrac{5000}{x}$

$x = \dfrac{5000}{\tan 40^o 32'} = \dfrac{5000}{0.8551} = 584736m$

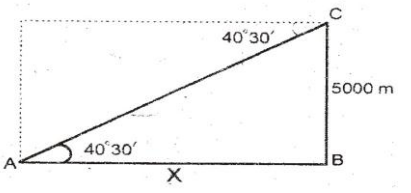


Fig. 6.12

Example 6: From a point on the ground the measure of angle of elevation of the top of tower is 30^o. On walking 100 meters towards the tower the measure of the angle is found to be of 45^o. Find the height of the tower.

Solution:

Let BC = height of tower

= h = ?

And DB = x m

AD = 100 m

AB = 100 + x

In right ΔABC

$\text{Tan}30^o = \dfrac{BC}{AB}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{100+x}$

$100 + x = \sqrt{3}\,h$ …… (1)

In right ΔBDC

$\tan 45^o = \dfrac{h}{x}$

$1 = \dfrac{h}{x}$

$x = h$ …….. (2)

Put x = h in (1)

$100 + h = \sqrt{3}\,h$

$1.7321h - h = 100$

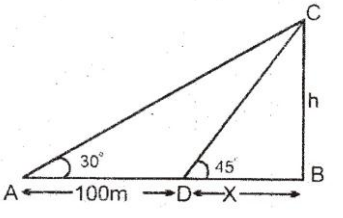


Fig. 6.13

$$h = \frac{100}{0.7321} = 136.60\text{m}$$

Example 7:

A pole being broken by the wind, its top struck ground at an angle of 30^o and at a distance of 10m from the foot of the pole. Find the whole height of the pole.

Solution:

Let BC = h = height of pole = ?

AD = CD

In right Δ ABD

$$\text{Tan}30^o = \frac{BD}{10}$$

$BD = 10\tan 30^o = 10(0.5774) = 5.77\text{m}$

Also

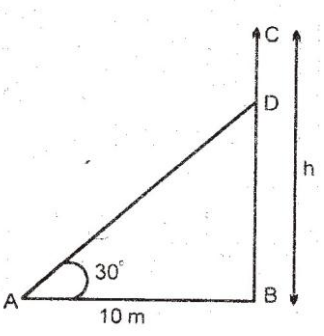


Fig. 6.14

$$\text{Cos}30^o = \frac{AB}{AD} \Rightarrow AD = \frac{10}{\text{Cos}30^o} = \frac{10}{0.8660} = 11.55\text{m}$$

Height of pole = h = BD + AD

$\therefore$ AD = CD

h = 11.55 + 5.77 = 17.32m

Exercise 7.2

Q1. How far is a man from the foot of tower 150 meters high, if the measure of the angle of elevation of its top as observed by him is $40^o\ 30'$.

Q2. The shadow of a building is 220 meters when the measure of the angle of elevation of the sun is 35^o. Find the height of the building.

Q3. The measure of the angle of elevation of a kite is 35. The string of the kite is 340 meters long. If the sag in the string is 10 meters. Find the height of the kite.

Q4. A man 18dm. tall observes that the angle of elevation of the top of a tree at a distance of 12m from the man is 32^0.What is the height of the tree?

Q5. On walking 300 meters towards a tower in a horizontal line through its base, the measure of the angle of elevation of the top changes from 30^o to 60^o. Find the height of the tower.

Q6. The measure of the angle of elevation of the top of a cliff is 25. On walking 100 meters straight towards the cliff, the measure of the angle of elevation of the top is 48^o. Find the height of the cliff.

Q7. From two points A and B. 50 meters apart and in the line with a tree, the measures of the angles of elevation of the top of the tree are 30^o and 40^o respectively. Find the height of the tree.

Q8. Two men on the opposite sides of a tower observe that the measures of the angles of elevation of the tower as observed by them separately are 15^o and 25^o respectively. If the height of the tower is 150 meters. Find the distance between the observers.

Q9. From a light-house, angles of depression of two ships on opposite of the light-house are observed to be 30^o and 45^o. If the height of the light house be 300m. Find the distance between the ships of the line joining them passes through foot of light-hosue.

Q10. The measure of angle elevation of the top of a tower is 30^o from a point on the ground. On retreating 100 meters, the measure of the angle of elevation is found to be 15^o. Find the height of the tower.

Q11. From the top of a hill 200 meters high, the angles of depression of the top and bottom of a tower are observed to be 30^o and 60^o respectively. Find the height of the tower.

Q12. A television antenna is on the roof of a building. From a point on the ground 36m from the building, the angle of elevation of the top and the bottom of the antenna are 51^o and 42^o respectively. How tall is the antenna?

Q13. A ladder 20 meter long reaches the distance of 20 meters, from the top of a building. At the foot of the ladder the measure of the angle of elevation of the top of the building is 60^o. Find the height of the building.

Q14. A man standing on the bank of a canal observes that the measure of the angle of elevation of a tree is 60^0.On retreating 40m from the bank , he finds the measure the angle of elevation of the tree as 30^0.Find the height of the tree and the width of the canal.

Q15. Two buildings A and B are 100m apart. The angle of elevation from the top of the building A to the top of the building B is 20^0. The angle of elevation from the base of the building B to the top of the building A is 50^0. Find the height of the building B.

Answers 7.2

(1)	175.63m	(2)	154.05m	(3)	h = 189.29m
(4)	9.6m	(5)	h = 259.81m	(6)	h = 80.37m
(7)	h = 17.10m	(8)	881.58m	(9)	819.6m
(10)	49.98m	(11)	133.3m	(12)	12.1m
(13)	h = 30m	(14)	34.64m ; 20m	(15)	155.5 m

7.4 Law of Sines:

In any triangle, the length of the sides are proportional to the sines of measures of the angle opposite to those sides. It means

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma}$$

Proof: Let one angle of the triangle say β be acute, then γ will be either acute, obtuse or right as in figure 1, 2, 3.

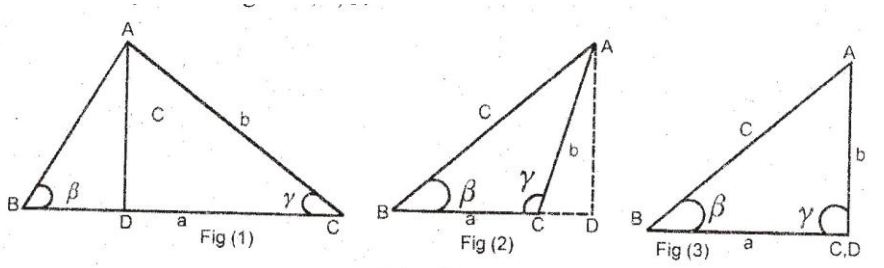


Fig. 6.15

Draw $AD \perp BC$ or BC produced.

Then from ΔABC (for all figures)

$$\frac{AD}{AB} = \text{Sin}\beta \qquad \therefore \ AD = c\text{Sin}\beta \ldots\ldots\ldots\ldots\ldots \quad (1)$$

If γ is acute in figure (1) $\dfrac{AD}{AC} = \text{Sin}\,\gamma \qquad \Rightarrow AD = b\ \text{Sin}\,\gamma$

If γ is obtuse in figure (2) $\dfrac{AD}{AC} = \text{Sin}\,(180 - \gamma) = \text{Sin}\,\gamma$

$$\Rightarrow AD = b\ \text{Sin}\,\gamma$$

If γ is right in figure (3) $\dfrac{AD}{AC} = 1 = \text{Sin}\ 90^{o} = \text{Sin}\,\gamma$

$$AD = b\ \text{Sin}\,\gamma$$

In each case we have

$$AD = b\ \text{Sin}\,\gamma \ldots\ldots\ldots\ldots \quad (2)$$

From (1) & (2), we have

It can similarly be proved that:

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta}, \text{ Similarly, } \frac{a}{\text{Sin}\alpha} = \frac{c}{\text{Sin}\,\gamma}$$

Hence ,

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma}$$

This is known as law of sines.

Note: we use sine formula when

i. one side and two angles are given

ii. two sides and the angle opposite one of them are given

Example 1:

In any ΔABC

a = 12, b = 7, $\alpha = 40^o$ Find β

Solution:

By law of sines $\frac{a}{Sin\alpha} = \frac{b}{Sin\beta} = \frac{c}{Sin\gamma}$

$$\Rightarrow \frac{a}{Sin\alpha} = \frac{b}{Sin\beta} = \frac{12}{Sin40^o} = \frac{7}{Sin\beta}$$

$$\Rightarrow Sin\beta = \frac{7Sin40^o}{12} = \frac{7(0.6429)}{12}$$

Sin β = 0.3750

$$\Rightarrow \beta = Sin^{-1}(.3750) \qquad \Rightarrow \beta = 22^o1'$$

Example 2:

In any ΔABC, b = 24, c = 16

Find the ratio of $Sin\beta$ to $Sin\gamma$

Solution:

By law of sines

$$\frac{a}{Sin\alpha} = \frac{b}{Sin\beta} = \frac{c}{Sin\gamma}$$

$$\Rightarrow \quad \frac{b}{Sin\beta} = \frac{c}{Sin\gamma} \Rightarrow \frac{Sin\beta}{Sin\gamma} = \frac{b}{c} = \frac{24}{16} = \frac{3}{2}$$

Example 3:

A town B is 15 km due North of a town A. The road from A to B runs North 27^o, East to G, then North 34^o, West to B. Find the distance by road from town A to B.

Solution:

Given that: c = 15 km $\alpha = 24^o$, $\beta = 34^o$

We have to find

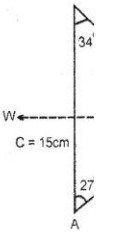
Distance from A to B by road.

Since $\alpha + \beta + \gamma = 180^o \quad \Rightarrow \quad 27^o + 34^o + \gamma = 180^o$

$$\gamma = 119^o$$

By law of sines:

$$\frac{a}{\text{Sin}\alpha}=\frac{b}{\text{Sin}\beta}=\frac{c}{\text{Sin}\gamma}$$



$$\Rightarrow \quad \frac{a}{\text{Sin}\alpha}=\frac{c}{\text{Sin}\gamma} \quad \Rightarrow \quad a=\frac{\text{Sin}\alpha}{\text{Sin}\gamma}$$

$$a=\frac{15\text{Sin}27^o}{\text{Sin}119^o}=\frac{15(0.4539)}{0.8746}=7.78$$

Fig. 7.16

Also $\frac{b}{\text{Sin}\beta}=\frac{c}{\text{Sin}\gamma} \quad \Rightarrow \quad \frac{b}{\text{Sin}34^o}=\frac{15}{\text{Sin}119^o}$

$$b=\frac{15\text{Sin}34^o}{\text{Sin}119^o}=\frac{15(0.5592)}{0.8746}=9.59$$

Thus distance from A to B by road:

$= b + a = 9.59 + 7.78 \qquad = 17.37\text{km}$

Exercise7.3

In any triangle ABC if:

Q1. $a = 10 \quad b = 15 \quad \beta = 50^o$ Find α

Q2. $a = 20 \quad c = 32 \quad \gamma = 70^o$ Find α

Q3. $a = 3 \quad b = 7 \quad \beta = 85^o$ Find α

Q4. $a = 5 \quad c = 6 \quad \alpha = 45^o$ Find γ

Q5. $a = 20\sqrt{3} \quad \alpha = 75^o \quad \gamma = 60^o$ Find c

Q6. $a = 211.3 \quad \beta = 48^o16' \quad \gamma = 71^o\,38'$ Find b

Q7. $a = 18 \quad \alpha = 47^o \quad \beta = 102^o$ Find c

Q8. $a = 475 \quad \beta = 72^o15' \quad \gamma = 43^o\,30'$ Find b

Q9. $a = 82 \quad \beta = 57^o \quad \gamma = 78^o$ Find a

Q10. $\alpha = 60^o \quad \beta = 45^o$ Find the ratio of b to c

Q11. Two shore batteries at A and B, 840 meters apart are firing at a target C. The measure of angle ABC is 80^o and the measure of angle BAC is 70^o. Find the measures of distance AC and BC.

Answers 7.3

1. $\alpha = 30^o\,42'\,37''$
2. $\alpha = 35^o\,37'\,58''$
3. $\alpha = 25^o\,16'\,24''$
4. $\gamma = 58^o\,3'$

5.	c = 31.06	6.	b= 181.89
7.	c = 12.68	8.	b = 449.22
9.	a = 69.13	10.	0.7319
11.	1578.68, 1654.46m		

7.5 The Law of Cosines:

This law states that "the square of any sides of a triangle is equal to the sum of the squares of the other two sides minus twice their product times the cosine of their included angle. That is

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = c^2 + a^2 - 2ac\cos\beta$$
and $$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Proof:

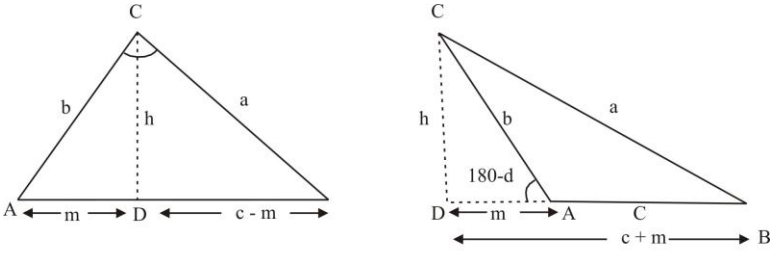


Fig. (I) Fig. (II)

Fig. 6.17

Let β be an acute angle of ΔABC, draw $CD \perp AB$

Let AD = m and CD = h

In right triangle BCD, we have

$$(BC)^2 = (BD)^2 + (CD)^2$$
$$a^2 = (BD)^2 + h^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

(i) If α is an acute angle, then from (i)

In right triangle ACD,

$$\text{Sin}\,\alpha = \frac{h}{b} \quad \Rightarrow h = b \text{ Sin } \alpha$$

and $$\text{Cos}\,\alpha = \frac{m}{b} \quad \Rightarrow m = b \text{ Cos } \alpha$$

So, $\boxed{BD = c - m = c - b\cos\alpha}$

Putting the values of h and BD in equation (1)

$$a^2 = (c - b\cos\alpha)^2 + (b\sin\alpha)^2$$
$$= c^2 - 2bc\cos\alpha + b^2\cos^2\alpha + b^2\sin^2\alpha$$
$$= c^2 - 2bc\cos\alpha + b^2(\cos^2\alpha + \sin^2\alpha)$$

$= c^2 - 2bc\cos\alpha + b^2$

$\boxed{a^2 = b^2 + c2 - 2bc\cos\alpha}$

(ii) If α in an obtuse angle, then from fig (ii)

In right triangle ACD,

$$\text{Sin}(180 - \alpha) = \frac{h}{b}$$

$$\text{Sin}\ \alpha = \frac{h}{b} \quad \Rightarrow h = b \sin\alpha$$

and $$\cos(180 - \alpha) = \frac{m}{b}$$

$$-\cos\alpha = \frac{m}{b} \quad \Rightarrow m = -b\cos\alpha$$

So, $\boxed{BD = c + m = c - b\cos\alpha}$

Putting the values of h and BD in equation (1)

$$a^2 = (c - b\cos\alpha)^2 + (b\sin\alpha)^2$$

we get, $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Similarly we obtain

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

and $$c^2 = b^2 + c^2 - 2bc\cos\alpha$$

Also when three sides are given, we find

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc},\ \text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac} \text{ and}$$

$$\text{Cos}\,\gamma = \frac{a^2+b^2-c^2}{2ab}$$

Note: we use the cosine formula, when

(i) Two sides and their included angle are given.

(ii) When the three sides are given.

Example 1: In any by using the law of cosines

$a = 7, c = 9, \beta = 112^0$ Find b

Solution: By law of cosines

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$b^2 = (7)^2 + (9)^2 - 2(7)(9)\ \text{Cos}\ 112^o$

$= 49 + 81 - 126(.3746)$

$b^2 = 130 + 47.20 = 177.2$

$b = 13.31$

Example 2: Two man start walking at the same time from a cross road, both walking at 4 km/hour. The roads make an angle of

measure 80^o with each other. How far apart will they be at the end of the two hours?

Solution: Let, A be the point of starting of two man V = 4 km/hour

Distance traveled by two men after 2 hours = vt

= 4 x 2 = 8km

Thus, we have to find BC = a = ?

By law of cosine:

$a^2 = b^2 + c^2 - 2bc \cos\alpha$

$a^2 = (8)^2 + (8)^2 - 2(8)(8) \text{ Cos } 80^o = 128 - 128\ (0.1736)$

$a^2 = 105.77 \quad \Rightarrow \quad a = 10.28\text{km}$

Thus, two men will apart 10.28 km after two hours.

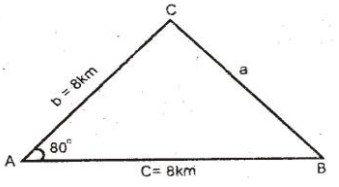


Fig. 6.18

Fig. 7.18

Exercise 7.4

In any triangle ABC by using the law of cosines:

1. a = 56 c = 30 $\beta = 35^o$ Find b
2. b = 25 c = 37 $\alpha = 65^o$ Find a
3. b = 5 c = 8 $\alpha = 60^o$ Find a
4. a = 212 c = 135 $\beta = 37^o\ 15'$ Find b
5. a = 16 b = 17 $\gamma = 25^o$ Find c
6. a = 44 b = 55 $\gamma = 114^o$ Find c
7. a = 13 b = 10 c = 17 Find α and β
8. Three villages P, Q and R are connected by straight roads. Measure PQ is 6 km and the measure QR is 9km. The measure of the angle between PQ and QR is 120^o. Find the distance between P and R.
9. Two points A and B are at distance 55 and 32 meters respectively from a point P. The measure of angle between AP and BP is 37^o. Find the distance between B and A.
10. Find the cosine of the smallest measure of an angle of a triangle with 12, 13 and 14 meters as the measures of its sides.

Answers 7.4

1. b = 35.83 2. a = 34.83 3. a =7
4. b = 132.652 5. c = 7.21 6. c = 83.24
7. $\alpha = 49^o\ 40'\ 47''$ $\beta = 35^o\ 54'\ 30''$
8. 13.08km 9. 35.18m 10. 52^o 37’

7.6 Solution of Oblique Triangles:

Definition:

The triangle in which have no right angle is called oblique triangle.

A triangle has six elements (i.e. three sides and three angles) if any three of a triangle are given, provided at least one of them is a side, the remaining three can be found by using the formula discussed in previous articles i.e. law of sines and law of cosines.

There are four important cases to solve oblique triangle.

Case I: Measure of one side and the measures of two angles.

Case II: Measure of two sides and the measures of the angle included by them.

Case III: When two sides and the angle opposite to one of them is given.

Case IV: Measure of the three sides.

Example 1:

Solve the ABC with given data.

$a = 850, \quad \alpha = 65^o, \quad \beta = 40^o$

Solution:

Given that:

$a = 850, \quad \alpha = 65^o, \quad \beta = 40^o$

$b = ? \quad c = ? \quad \gamma = ?$

Since, $\alpha + \beta + \gamma = 180^o$

$65^o + 40^o + \gamma = 180 \quad \gamma \quad \Rightarrow \gamma = 75^o$

By law of sines to find b:

$$\frac{a}{\text{Sin}\,\alpha} = \frac{b}{\text{Sin}\,\beta}$$

$$\frac{850}{\text{Sin}\,65^o} = \frac{b}{\text{Sin}\,40^o} \Rightarrow \quad b = \frac{850\ \text{Sin}\,40^o}{\text{Sin}\,65^o}$$

$$b = \frac{850(0.64.28)}{0.9063} = 602.85$$

To find c, by law of Sines

$$\frac{b}{\text{Sin}\,\beta} = \frac{a}{\text{Sin}\,\alpha} \quad \Rightarrow c = \frac{b\ \text{Sin}\,\gamma}{\text{Sin}\,\beta}$$

$$c = \frac{602.85\ \text{Sin}\,75^o}{\text{Sin}\,40^o} = \frac{602.85(0.9659)}{0.6428}$$

$= 905.90$

Example2:

Solve the triangle with given data:

$a = 45 \quad b = 34 \quad \gamma = 52^o$

Solution:

Given a = 45 b = 34 $\gamma = 52^o$

c = ? β = ? γ = ?

To find c, we use law of cosines

$c^2 = a^2 + b^2 - 2ab \cos \gamma$

$c^2 = (45)^2 + (34)^2 - 2(45)(34) \cos 52^o$

$c^2 = 2025 + 1156 - (3060)(0.6157)$

$c^2 = 1297 \quad \Rightarrow \quad \boxed{c = 36.01}$

To find α, we use law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\gamma} \Rightarrow \frac{45}{\sin\alpha} = \frac{36.01}{\sin 52^o}$$

$$\sin\alpha = \frac{45 \sin 52^o}{36.01} = \frac{45(0.7880)}{36.01} = 0.9847$$

$\alpha = \sin^{-1}(0.9847) = 97^o\ 58'\ 39''$

To find β, we use $\alpha + \beta + \gamma = 180^o$

$79^o\ 58'\ 39'' + \beta + 52^o = 180^o$

$\beta = 48^o\ 01'\ 20''$

Exercise 7.5

Solve the triangle ABC with given data.

Q1. c = 4 $\alpha = 70^o$ $\gamma = 42^o$

Q2. a = 464 $\beta = 102^o$ $\gamma = 23^o$

Q3. b = 85 $\beta = 57^o 15'$ $\gamma = 78^o 18'$

Q4. b = 56.8 $\alpha = 79^o 31'$ $\beta = 44^o 24'$

Q5. b = 34.57 $\alpha = 62^o 11'$ $\beta = 63^o 22'$

Q 6. Find the angle of largest measure in the triangle ABC where:

(i) a = 224 b = 380 c = 340

(ii) a = 374 b = 514 c = 425

Q7. solve the triangle ABC where:

(i) a = 74 b = 52 c = 47

(ii) a = 7 b = 9 c = 7

(iii) a = 2.3 b = 1.5 c = 2.7

Answers 7.4

Q1. $a = 5.62$ $b = 5.54$ $\beta = 68^o$

Q2. $\alpha = 55^o$ $b = 454$ $c = 221.31$

Q3. $\alpha = 44^o 27'$ $a = 70.78$ $c = 98.97$

Q4. $a = 79.82$ $c = 67.37$ $\gamma = 56^o 0'$

Q5. $a = 34.20$ $c = 31.47$ $\gamma = 54^o 2'$

Q6. (i) $81^o 55' 57''$ (ii) $79^o 47' 53''$

Q7. (i) $\alpha = 96^o 37'$ $\beta = 44^o 16'$ $\gamma = 39^o 07'$

(ii) $\alpha = 50^o$ $\beta = 80^o$ $\gamma = 50^o$

(iii) $\alpha = 58^o 21'$ $\beta = 33^o 45'$ $\gamma = 87^o 55'$

Summary

1. Right Triangle:

A triangle which has one angle given equal to a right angle.

2. Oblique Triangle:

The triangle in which have no right angle is called oblique triangle.

3. Law of Sines

In any ΔABC, the measures of the sides are proportional to the sines of the opposite angles.

i.e. $\dfrac{a}{\text{Sin}\alpha} = \dfrac{b}{\text{Sin}\beta} = \dfrac{c}{\text{Sin}\gamma}$

4. Law of Cosines

(i) $a^2 = b^2 + c^2 - 2bc \text{ Cos } \alpha$

(ii) $b^2 = a^2 + c^2 - 2ac \text{ Cos } \beta$

(iii) $c^2 = a^2 + b^2 - 2ab \text{ Cos } \gamma$

(iv) $\text{Cos } \alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

(v) $\text{Cos } \beta = \dfrac{a^2 + c^2 - b^2}{2ac}$

(vi) $\text{Cos } \gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$

Angle of Elevation:

The angle AOP which the ray from an observer's eye at O to an object P at O to an object P at a higher level, makes with horizontal ray OA through O is called the angle of elevation.

Angle of Depression:

The angle AOP which the ray from an observer's eye at O to an object at P at a lower level makes with the horizontal ray OA through O is called the angle of depression.

Short Questions

Write the short answers of the following:

Q.1: Define the law of sine.

Q.2: Define the Laws of cosines

Q.3: In right triangle ABC, $\gamma = 90^o$, a = 5, c = 13 then find the value of angle α .

Q.4: Given that $\gamma = 90^o$, $\alpha = 35^o$, a = 5, find angle β

Q.5: In right triangle ABC b = 6, $\alpha = 35^o$, $\gamma = 90^o$, Find side 'a'

Q.6: Given that $\alpha = 30^o$, $\gamma = 135^o$, and c = 10 , find a

Q.7: In any triangle ABC, if a =20, c = 32 and c =70^o, Find A.

Q.8: In any triangle ABC if a = 9, b = 5, and $\gamma = 32^o$, Find c.

Q.9: The sides of a triangle are 16, 20 and 33 meters respectively. Find its greatest angle.

Q.10: Define angle of elevation and depression.

Q.11: A string of a flying kite is 200 meters long, and its angle of elevation is 60^o. Find the height of the kite above the ground taking the string to be fully stretched.

Q.12: A minaret stands on the horizontal ground. A man on the ground, 100 m from the minaret, Find the angle of elevation of the top of the minaret to be 60^o. Find its height.

Q.13: The shadow of Qutab Minar is 81m long when the measure of the angel of elevation of the sun is $41^o\ 31'$. Find the height of the Qutab Minar.

Q.14: In any triangle ABC in which

b = 45, c =34, $\alpha = 52^o$, find a

Q.15: In any triangle ABC is which

a = 16, b = 17, $\gamma = 25^o$ find c

Q.16: In any triangle ABC in which

a = 5, c =6, $\alpha = 45^o$ Find sin γ

Q.17: b =25, c =37 a =65^o find a

Q.18: $a = 16, b = 17, \quad \gamma = 25^o$ find c

Q.19: $a = 3, \quad b = 7, \quad \beta = 85^o$, Find α .

Answers

3. $22^o\,37'$. 4. $\beta = 55^o$ 5. $a = 4.2$ 6. $a = 7.07$

7. $A = 35^o\,77'\,58''$ 8. $c = 5.48$ 9. $\gamma = 132^o\,34'$ 11. $h = 173.2$ m

12. $h = 173.20$ m 13. $h = 71.66$ m 14. $a = 36.04$ 15. $c = 7.21$

16. $\gamma = 58^o\,3'$ 17. $a = 34.82$ 18 $c = 7.21$ 19 $\alpha = 25^o 14' 14''$

Objective Type Questions

Q.1 Each questions has four possible answers. Choose the correct answer and encircle it.

__1. Law of sines is:

(a) $\frac{a}{\sin B}=\frac{b}{\sin A}=\frac{c}{\sin c}$ (b) $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

(c) $\frac{a}{\sin B}=\frac{b}{\sin A}=\frac{c}{\sin C}$ (d) $\frac{a}{\sin B}=\frac{b}{\sin C}=\frac{a}{\sin A}$

__2. In a triangle ABC $\angle A = 70^o$, $\angle B = 60^o$, then $\angle C$ is:

(a) 30^o (b) 40^o (c) 50^o (d) 60^o

__3. When angle of elevation is viewed by an observer, the object is:

(a) Above (b) Below

(c) At the same level (d) None of these

__4. If b = 2, $A = 30^o$, $B = 45^o$, then a is equal to:

(a) 2 (b) $\sqrt{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

__5. If a = 2, b = 2, $A = 30^o$, then B^o is:

(a) 45^o (b) 30^o (c) 60^o (d) 90^o

__6. If in a triangle ABC, the sides b, c and angle A are given, then the side a is:

(a) $a^2 = b^2 + c^2 + 2bc \cos A$ (b) $a^2 = b^2 - c^2 - 2ab \cos A$

(c) $a^2 = b^2 + c^2 - 2bc \cos A$ (d) $a^2 = b^2 - c^2 + 2ab \cos A$

__7. In a triangle ABC, the law of cosine is:

(a) $\text{Cos A}= \frac{b^2+c^2+a^2}{2bc}$ (b) $\text{Cos A}= \frac{b^2+c^2-a^2}{2bc}$

(c) $\text{Cos A}= \frac{b^2+c^2+a^2}{2ab}$ (d) $\text{Cos A}= \frac{b^2+c^2-a^2}{2ac}$

__8. If in a triangle ABC, b = 2, c = 2, $A = 60^o$, then side a is:

(a) 2 (b) 3 (c) 4 (d) 5

__9. If in a triangle ABC, a = 1, b = $\sqrt{2}$, $C = 60^o$, then side c is:

(a) $\sqrt{2}$ (b) 2 (c) 1 (d) 3

__10. If in a triangle ABC, b = 2, c = 3, a = 1, then cos A is:

(a) 1 (b) 2 (c) 3 (d) 4

__11. If in a triangle ABC, a = 3, b = 4, c = 2, then cos C is:

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{7}{8}$ (d) 3

__12. If b sin C = c sin B, then, a sin B is equal to:

(a) c sin A (b) b sin c (c) b sin A (d) b sin B

__13. In a right triangle if one angle is 30^o, then the other will be:
(a) 45^o (b) 50^o (c) 60^o (d) 75^o

__14. In a right triangle if one angle is 45^o, then the other will be:
(a) 45^o (b) 50^o (c) 60^o (d) 75^o

__15. If $B = 90^o$, $b = 2$, $A = 30^o$, then side a is:
(a) 4 (b) 3 (c) 2 (d) 1

__16. If $c = 90^o$, $a = 1$, $c = 2$, then angle A is:
(a) 90^o (b) 60^o (c) 45^o (d) 30^o

__17. If $c = 90^o$, $b = 1$, $c = \sqrt{2}$, then side a is:
(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 3

__18. If $c = 90^o$, $b = 1$, $c = \sqrt{2}$, then angle A is:
(a) 15^o (b) 30^o (c) 45^o (d) 60^o

__19. The distance of a man from the foot of a tower, 100m high if the angle of elevation of its top as observed by the man is 30^o is:
(a) 50m (b) 100m (c) 150m (d) 200m

__20. A pilot at a distance of 50m, measure the angle of depression of a tower 30^o, how far is the plane from the tower:
(a) 50m (b) 25m (c) 20m (d) 10m

Answers

Q1:

1.	b	2.	c	3.	a	4.	d	5.	b
6.	c	7.	b	8.	a	9.	c	10.	a
11.	c	12.	c	13.	c	14.	a	15.	d
16.	d	17.	a	18.	c	19.	d	20.	b